

# Cooperation and emergence of role differentiation in the dynamics of social networks \*

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By means of extensive computer simulations, the authors consider the entangled coevolution of actions and social structure in a new version of a spatial Prisoners Dilemma model that naturally gives way to a process of social differentiation. Diverse social roles emerge from the dynamics of the system: leaders are individuals getting a large payoff who are imitated by a considerable fraction of the population, conformists are unsatisfied cooperative agents that keep cooperating, and exploiters are defectors with a payoff larger than the average one obtained by cooperators. The dynamics generate a social network that can have the topology of a small world network. The network has a strong hierarchical structure in which the leaders play an essential role in sustaining a highly cooperative stable regime. But disruptions affecting leaders produce social crises described as dynamical cascades that propagate through the network.

## I. INTRODUCTION

Social traps (Platt 1973) are situations in which rational individual choices result in an undesirable collective outcome for the social group. A well known example is the problem of establishing cooperation in a social group (Axelrod 1994) or, more generally, problems of public goods (Olson 1965, Hardin 1968). It is often recognized that the understanding of collective social behavior, once individual attitudes are known, requires taking into account the interactions among the individuals of the group, and that these are mediated by a network of social relations (Granovetter 1973, 1978). Such network constitutes the social structure of the group. The embeddedness of the interactions in the social structure (Granovetter 1985) has been identified as a main ingredient in explaining the evolution of cooperation (Macy and Skvoretz 1998). It is also well documented (Lazer 2001), that in the same way that the actions of the individuals are affected by the social network, the network is not an exogenous structure, but it is rather created by individual choices. However, there are not many specific models of social dynamics that incorporate explicitly the concept of co-evolution of individual and network (Lazer 2001). In fact, in the long term research agenda posed by Macy (1991) a central point is that the structure of the network should not be considered as given, but as a variable. The question posed is *...to explore how social structure might evolve in tandem with the collective action it makes possible*. This goes further beyond models in which there is

some network evolution decoupled from the evolution of the actions of the individuals in the group. In the context of reciprocal altruism and the building of cooperation, this general question was implicitly considered within a game theory simulation model named the Social Evolution Model (SEM) (de Vos et al., 2001; Zeggelink et al., 2000). In this paper we address explicitly the problem of co-evolution in a version of the Prisoner's Dilemma (PD) (Rapoport and Chammah 1965) in which players interact through a network that adapts to the results of the game, and therefore to the actions of the players. We focus on the resulting type of social structure (cohesive group versus social hierarchy), as well as on the dynamical mechanisms needed to produce the topological properties of the network of interactions that stabilize a collective cooperative behavior.

A main result of our analysis, extensively based on computer simulations (Zimmermann et al. 2001), is the *emergence* of a process of social differentiation together with the building-up of a network with hierarchical relations. Starting from random partnership among equivalent individuals, a social structure emerges. In this emerging structure, the topology of the network of social relations identifies individual with different social roles: leaders, conformists and exploiters. These roles have been spontaneously selected during the complex adaptive evolution of the social group that entails a learning process. This social structure sustains global cooperation with exploiters surviving in hierarchical chains in the network. This result is at variance with other simulation models, such as SEM, in which partner selection leads to the formation of cohesive egalitarian clusters (de Vos et al., 2001; Zeggelink et al., 2000): the emergence of these groups in a socially segmented population, with exclusion of free riders, seems to be the basis of the survival of cooperation in these other studies. Concerning the topology of the co-evolved network that sustains

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cooperation, we find a hierarchical network with an exponential connectivity distribution. Our results show that clustering is not needed to sustain cooperation. However the additional inclusion of local neighboring partner selection in the model generates the celebrated small-world connectivity (Watts 1999) of the network.

From a philosophical point of view, the concept of *emergence* is used in contemporary sociology in contradictory ways, being its proper meaning debatable (Sawyer 2001). We use it here as it appears in multi-agent models of social systems (Gilbert and Conte 1965; Schelling 1978; Epstein and Axtell 1996; Axelrod 1997). In this context it refers, as often also used in the natural sciences, to complex dynamical behavior or properties that cannot be reduced to, or predicted from a detailed description of the units that compose a system, so that the reductionist hypothesis does not imply a constructionist one (Anderson 1972). These ideas put forward a hierarchical structure of science in which different concepts and descriptions are needed at different levels. The key idea is that sociology cannot be reduced to psychology, as molecular biology is not just applied chemistry (Anderson 1972). From this perspective, and recognizing the limitations of game theory in describing human interactions, we learn much from a metaphor like the Prisoner's Dilemma when considering emergent properties which are not simply linked to special features of individual human behavior.

A canonical example of emergence is the V shape of bird flocks (Sawyer 2001). The shape of the flock and the fact that a particular bird plays the role of a leader, with other birds lining up behind it, is a nontrivial result of simple interaction rules, but the leader is neither genetically determined nor externally appointed. Our results parallel this example, showing how equivalent agents confronted with the choice of an action (cooperating or defecting) and with the possibility of choosing partners, differentiate and acquire different roles while building up a social structure. This process is the result of interactions among neighbors that determine an entangled co-evolution of the choice of actions and of the social network. A particular enlightening example of the general process that we address here is the cooperative relations among researchers in a given scientific field. A given researcher might choose to work or not to work together with other scientist and, depending on the degree of success of this collaboration, he might search in the community to find other scientists with which a profitable cooperative relation can be alternatively established. This co-evolution process results in a network of collaborations in which different scientists play different roles.

Our results might be compared with other studies of group formation beyond those associated with the SEM. In the theory of group stability of Carley (1991), as it also happens in our simple model of equivalent agents, individuals assume multiple roles. Also there, these diverse roles produce, at the group level, group stability or change depending on the social structure. However,

while the theory of Carley is based on sharing of knowledge, this aspect is missing in our model. Stability is in our case strictly dependent on the adapting structure of the network of interactions.

In the remainder of this introduction we give a brief discussion of the main ingredients of our analysis. These ingredients are models of cooperation, models of social networks, and the implications of the will of the agents that manifests itself in the ability of making choices. The second section of the paper presents our model and some general conclusions from its analysis. The main body of our computer simulation results are summarized in Section 3. We conclude with a discussion of results and limitations of our study.

### A. Routes to cooperation

Why do people cooperate? Why cooperation is empirically observed when there is a conflict between the self-interest and the common good? Classical answers to these questions are formulated in terms of kin (Hamilton 1964) or group selection (Wilson and Sober 1994), or cooperation based upon reciprocity (Trivers 1971, Axelrod and Hamilton 1981, Axelrod 1984). The first answers are biologically inspired on the increased biological fitness of kinship or on the adaptive success of groups of Cooperators. Still, the idea of social kin selection has been put forward (Riolo et al. 2001): cooperation can arise from similarity, and tags that identify such similarity can be based in cultural attributes instead of being genetically determined. Indeed, cultural transmission has been invoked as a potential reason for the prevalence of cooperation in human populations (Mark 2002). Other route to cooperation, based on a reputation score for each individual and information sharing (Nowak and Sigmund 1998), also emphasizes the cultural forces present in human society. Cooperation grounded on reputation can be seen however, as a form of indirect reciprocity.

Cooperation based upon reciprocity is often formalized through the iterated Prisoner's Dilemma (Rapoport and Chammah 1965). The game theoretical formulation of the PD shows that two perfectly rational agents interacting once, and confronted with the choice of cooperating or defecting, would both choose to defect, which corresponds to the Nash equilibrium of the game. When the game is repeated or iterated, the Folk theorem classifies the many possible outcomes that can be sustained, in particular full cooperation. It is the *shadow of the future* in the repeated interaction that makes cooperation sustainable (Axelrod 1984). Although basic concepts and results on the PD were well known in game theory (Binmore 1998), the evolutionary ideas for the selection of an equilibrium pioneered by Axelrod (1984, 1997) have been extremely influential. They have lead to the consideration of the virtue of different strategies and to the concept of evolutionary stable strategies. It is now well accepted that dynamical models of cultural evolution and social

learning hold a greater chance for success than models merely based on rational choice. This body of knowledge has been reviewed by Hoffmann (2000), while some recent advances in the theory have been reviewed by Axelrod (2000).

Generally speaking, the ingredient of locality is not taken into account in the different studies of models of cooperation mentioned so far. Individuals or players interact globally by continuous changes of random pairings among them. However, local spatial interactions introduce another possibility of reciprocity which is not based on history dependent or backward-looking strategies. In this line of thinking, spatial PD games have been introduced (Axelrod 1984, Nowak and May 1992), and further generalized by Lindgren and Nordahl (1994). Spatial games and local interactions were also introduced in economic contexts by Blume (1993) and Ellison (1993). Nowak and May (1992, 1993) and Nowak et al. (1994a, 1994b) start considering a group of individuals placed at the nodes of a regular two dimensional square lattice. What they showed is that cooperation might arise from spatially distributed interactions: if agents play only with a local neighborhood in the lattice, clusters of Cooperators may survive the interaction with Defectors, and cooperation may be sustained. This can happen bypassing any consideration on memory and strategies, in situations in which non-cooperative behavior would prevail in the global game. Admittedly, the game theory formulation of cooperation neglects interpersonal relations driven by emotional processes (Lawler and Yoon 1998). In spatial PD games, the fundamental relationship is that of exchange conditioned by structural position. Still, the basic mechanism of imitation of successful neighbors is introduced. This mechanism is certainly prevalent in many human interactions. We finally mention that the analysis of spatial games, specially when searching for collective emergent behavior in societies with a large number of individuals, require the use of intensive computer simulations. There are however some analytical results (Schweitzer 2002), specially in simplified one-dimensional models (Eshel et al. 1998).

There are other routes to cooperation, among which we can mention optional participation (Hauert et al. 2002), and stochastic collusion (Macy 1991). Optional or volunteering participation is a mechanism that incorporates, in addition of Cooperators and Defectors, players (*loners*) that refuse to participate in the game. This has been shown to be an effective mechanism to escape from the social dilemma without invoking any form of reciprocity. Incorporating locality and spatial interactions, Cooperators also tend to fare better (Szabo and Hauert 2002). Stochastic collusion stems from a forward-looking route to learning and adaptive behavior (Macy and Flache 2002): stochastic search by adaptive individuals in response to immediate outcomes allows escaping from the non-cooperative social trap. The beneficial implications of locality (*bounded social space*) to sustain cooperation have also been discussed for this mechanism (Macy 1991).

## B. Social networks

A new field of opportunities for modeling is opened when considering the network of social interactions - links between individuals are not established in a random way, but depending on neighborhood or interest relationships. As a consequence, clusters and stable linkage are developed giving way to new aspects of cooperation in which interactions among individuals have an important effect, being crucial to the performance of the system as a whole (Holland et al 1986).

The consideration of the spatial version of the PD discussed above is a step in the direction of considering a social network. But a social network is in general different from a regular two dimensional lattice, as much as social interactions are different from a continuous random pairing of individuals. The popular phenomenon of *six degrees of separation* (Guare 1990), that follows from the early experiments of Milgram (1967), makes clear that if the average separation between two individuals is given by only six intermediate acquaintances (six links), social networks radically differ from the regular ones often considered in spatial games. Such small world effect has been re-popularized by Watts and Strogatz (1998). Beyond the physical and topological properties of the network, it is also important to recall that, in economic terms, a social network has been associated with a social capital (Coleman 1988), in the sense that it gives the basis for stable repeated social interactions.

The spatial version of the PD has been revisited by Axelrod and coworkers (Axelrod et al. 2000, Cohen et al. 2001) trying to understand the role of social structure and geographically based networks in the maintenance of cooperation. They have shown that geographically dispersed social networks are efficient in maintaining cooperation, provided the links are stable. They conclude therefore that the important ingredient is not clustering (i.e., locality or correlation of linkage patterns), but what they label as context preservation, that is the continuity of interactions. Another interesting contribution in this context is the study of Buskens and Weesie (2000) on the role of social structure in supporting cooperation via reputation. By including a social structure, this study goes beyond the ones of Nowak and May (1998) and Riolo et al. (2001) and it shows (Axelrod 2000) that these two aspects, social structure and reputation, reinforce each other in building up a cooperative collective behavior. Reputation established via information sharing leads to sustained cooperation in social structures that are less rigid than the ones fixed by geographic positions, adding therefore a higher degree of freedom.

From the early mathematical analysis of random networks (Erdos and Renyi, 1959) there is a fast growing literature devoted to uncovering the topological properties of technological, biological and social networks (Wasserman and Faust 1984, Watts and Strogatz 1998, Watts 1999a, Watts 1999b, Barabasi and Albert 1999, Amaral et al. 2000, Newman 2001, Newman and Strogatz

2001), partially reviewed by Albert and Barabási (2002), Barabási (2002), and Watts (2003). Strogatz (2001) emphasized that, despite their different nature, many networks present similar characteristics, but it is important to note that differences do also exist. The main properties of a network usually analyzed are average distance between nodes, clustering and degree distribution. A short average distance, that is, the fact that the number of steps needed to connect any pair of individuals in a social network is small, is what originally characterizes a small world effect. But it was long ago recognized (Wasserman and Faust 1984) that social networks also show a high tendency to form cliques, i.e., friends of friends are also friends. Such high clustering property and a small path length characterize the small world networks formalized by Watts (1999). These *small world networks* are half-way between random and regular networks. The degree distribution is the probability distribution of the number of links or connections of a node of the network. Many networks have a power law distribution, which implies a scale free distribution of degree (Barabasi and Albert 1999). However, it has been reported (Amaral et al. 2000) that social networks do not generally display such heterogeneity in their connectivity, but instead show a more homogeneous degree distribution, with the number of connections being characterized by a narrow distribution around a single scale.

Our concern in this paper is not the characterization of the topological properties obtained considering a snapshot of a social network. Rather, the question addressed is how the network is dynamically formed or how a given network structure is reached after social agents interact for a long time (Zimmermann et al. 2004). As Watts indicates (Watts 1999), networks affect the dynamics of the system in a passive and an active way. Examples of the passive way are the spatial version of the PD game and the variants thereof previously mentioned, or the study of a PD game in small world network (Abramson and Kuperman 2001). We are here interested in the active aspect in which the network of connections evolves by the will of the agents. We seek to unveil possible dynamical mechanisms to achieve a small world connectivity.

### C. Making choices and beyond: Spontaneous social differentiation

Incorporating the will of the agents seems to be a crucial ingredient in any model of human behavior. In the setting of a spatial version of the PD game in a social network, the agents should have some way of choosing their actions (cooperate or defect) and choosing their partners. Making choices is an instrument for learning in an adaptive evolution.

Partner and action selection has been taken into account in the iterated PD as reviewed by de Vos et al. (2001), but without considering local spatial interactions and also without reference to a social network. The vol-

unteering mechanism of Hauert et al. (2002) does not give a choice of action, since agents refusing to participate are fixed from the outset, but it is an indirect mechanism to determine who interacts with whom. More closely related to changes in the network structure, or to the choice of partners, is allowing players to exit from an unsatisfactory relationship with partners, as discussed by Axelrod (2000) on the grounds of the Edk-Group's analysis of a set of fifteen strategies related with the possibility of opting out (Edk-Group 2000). The possibility of exiting generally reinforces cooperation, but again this study does not include local interactions and social networks.

The formation of a social network based on individual decisions has been studied by Bala and Goyal (2000), but differently of our interest here, the individuals forming the network do not have some dynamics on top of the network of interactions: there is no game mediated by the interactions defined by the network. Skyrms and Pemantle (2000) do consider the simultaneous evolution of actions and structure of the network (fluid network), emphasizing that these type of models are largely unexplored. A generic result from their study is that important changes of global behavior occur when moving from frozen to fluid social networks. Their approach is similar to the one also considered by Macy (1991): Starting from a random network, the network is regenerated at each time step of the game by random pairings among the agents, but the probabilities of a pairing depend on the previous results of the game. This a basic idea of co-evolution, but it does not take into account aspects of locality, or a social network, that albeit evolving, has well established stable links among individuals.

Our starting point is the observation that the influence of social structure in human behavior, with feedback between actions and social structure, might be seen as an entangled co-evolution of the choice of actions and the formation of a social structure. Specifically, we address here the issue of the evolution of social networks in the context of cooperation, allowing individuals both to choose actions and to choose exiting from unsatisfactory relationships. We propose a simple model where the actions of individuals that form the social network are driven by their level of satisfaction, choosing their actions by imitation of their best neighbor. Moreover, we introduce *social plasticity*—other definitions are introduced by Lazer (2001)—as the capacity of the individuals to choose partners, being able to change their neighborhood as time goes on. Evolving interactions, thus, appear, allowing individuals to choose different partners on the grounds of the obtained performance.

The significant step forward in the results of our approach is that it naturally leads to a process of social differentiation. Starting from random partnership among equivalent individuals, a social structure emerges. In this emerging structure the topology of the network of social relations identifies individuals with different social roles. These roles have been spontaneously selected during the complex adaptive dynamics of the social group. They are

not the direct consequence of initial differences in strategy or location. Rather they emerge in a probabilistic dynamics in which the social network is constructed.

## II. PRISONER'S DILEMMA IN AN ADAPTIVE NETWORK

The simplest form of the PD game consists of two agents which may choose from either of two actions: to cooperate (C) or to defect (D). If both agents choose C, each agent gets a payoff (reward)  $R$ ; if one defects while the other cooperates, the former gets a payoff  $T$  (with  $T > R$ ), while the latter gets the "suckers" payoff  $S$  (with  $S < R$ ); if both defect, both get a payoff  $P$ . Under the standard restrictions  $T > R > P > S$ ,  $T + P < 2R$ , defection is the best choice in a single shot game (Nash equilibrium). Thus the social dilemma of how cooperation may be sustained arises, due to the fact that rational agents would defect. When no social structure is assumed, agents are drawn randomly in pairs to play the game, and the dynamics may be described by a replicator type equation (Hofbauer and Sigmund, 1998). This equation can be understood from the biologically motivated fact that strategies with fitness larger than the average in the whole population, replicate with a positive rate, while those that underperform the average, die away. In our context, if  $\Pi_C$  is the average payoff obtained using strategy C, and  $\langle \Pi \rangle$  is the average payoff for all the population, then the time evolution of the fraction of the agents using strategy C, labeled  $f_C$ , obeys the equation

$$\frac{df_C}{dt} = f_C(\Pi_C - \langle \Pi \rangle). \quad (1)$$

Given the payoffs of the PD game we obtain

$$\Pi_C = Rf_C + (1 - f_C)S \quad (2)$$

$$\langle \Pi \rangle = Rf_C^2 + (S + T)f_C(1 - f_C) + P(1 - f_C)^2, \quad (3)$$

and thus

$$\frac{df_C}{dt} = f_C(1 - f_C)((R - T)f_C + (S - P)(1 - f_C)). \quad (4)$$

The analysis of this equation indicates that it has two equilibrium solutions in  $f_C \in [0, 1]$ :  $f_C = 0$  is a stable fixed point, while  $f_C = 1$  is an unstable fixed point. Thus, it also follows from this dynamical analysis that in the absence of an interaction network describing a social structure the only stable solution is a pure defective state.

In the spatial version of the PD (Axelrod, 1984; Nowak et al., 1994a,b; Nowak and May, 1992, 1993) agents sit on the nodes of the network and they play the game with their neighbors. Two agents are said to be 'neighbors' if they are directly connected by a link of the network. A detailed analysis of the different spatiotemporal patterns that can arise in the dynamics shows a phase space where Cooperators and Defectors can coexist depending on the parameter values

((Lindgren and Nordahl, 1994; Schweitzer et al., 2002)). There are threshold values in parameter space beyond which defection dominates the whole network.

We will consider a spatial version of the PD, generalized in order to consider, instead of fixed, regular lattices, *adapting evolving networks* of interaction.

### A. Social plasticity

We want to study how the results obtained with fixed interaction networks change if certain *social plasticity* is allowed. This term addresses the common observation that social interactions in most societies adapt in time by a learning process. We will implement a rule for social plasticity which depends on both the strategies and pay-off of the individual players. The motivation of this rule comes from the following analysis of the mutual benefit obtained from each pair of allowed PD interactions: two symmetric (C-C and D-D) and one asymmetric (C-D). Clearly the symmetric interaction C-C should be reinforced because both agents get the maximum payoff from their selected strategies. However, the opposite occurs in a D-D interaction, where both agents have aligned incentives to change neighbor and to possibly find a C-neighbor. The asymmetric interaction C-D forms an intermediate class, where the C agent will not support the interaction, but the D agent will try to reinforce it. As a first approximation to the problem, we assume that this type of interaction does not change, because the overall effect of both agents is balanced. From this analysis, we conclude that the simplest non-trivial social plasticity rule should allow interactions among two D-agents to adapt, providing a self-interest mechanism of social adaptation where the agents can increase their payoffs by changing their partners.

### B. The model

We consider a fixed population of agents placed in the nodes of a network and connected by links to their neighbors. The dynamics evolves in discrete time steps divided in three stages, starting from a random choice of strategies and a random network:

1. *Interacting*: Each agent  $i$  plays the PD game with its neighbors, and collects an aggregate payoff  $\Pi_i$ ;
2. *Strategy update*: Each agent updates its current strategy, imitating the strategy of the neighbor with the largest payoff including himself (whenever more than one equivalent neighbor with a larger payoff exists, one of them is randomly selected);
3. *Neighborhood update*: If agent  $i$  imitates a Defector, then the agent  $i$  replaces with probability  $p$  this link with the imitated D-neighbor by a new one

pointing to a randomly chosen partner from the whole network. This process updates the network.

In order to clarify the rules of the model we would like to remark the following points. First, we specify that in the interaction step we only consider bidirectional (undirected) links, so if agent  $i$  plays with  $j$  then we also assume that  $j$  plays with  $i$ . Second, we do not consider complex strategies which involve the history of past encounters with neighbors (as in Lindgren and Nordahl (1994)), but we consider instead only zero-memory strategies, and assume that each agent plays the same strategy (action) C or D with all its partners. Third, the game is played synchronously, i.e., at each discrete time step agents decide their strategy in advance and they all play at the same time.

We consider in Fig. 1 an example that illustrates the rules of the model. The system is initially formed by 5 Cooperators (agents  $a$  to  $e$ ) and 1 Defector (agent  $f$ ). According to the payoff they obtain, agents  $a$ ,  $d$  and  $e$  are unsatisfied and they will imitate in the next time step the Cooperator  $c$ . Agents  $c$  and  $f$  are satisfied because they don't have a neighbor with a larger payoff. Finally agent  $b$  is also unsatisfied but the agent with largest payoff is the Defector  $f$ . Thus in the next time step it changes its strategy to become a Defector, and (with probability  $p$ ) breaks the link with agent  $f$  and selects a new neighbor as partner (in this case he selects agent  $a$ ). Note that in the neighborhood update rule it is not needed that an agent changes its strategy from Cooperator to Defector to sever a link. This example shows that the neighborhood update rule facilitates the survival of the Defectors by increasing their payoff when a Cooperator is selected at random.

The strategy and neighborhood update rules implemented in this model represent a useful choice to unveil basic mechanisms behind global cooperative behavior. Other choices are discussed later in Section 4. On the one hand, the extreme learning (imitation) rule of the strategy update can be justified by the psychological bias to focus on confirmation and neglect disconfirmation of beliefs (Strang and Macy, 2001). Bounded rational agents seek to learn from limited and biased information: they respond to perceived failure by imitating their most successful peer. In our case example of scientific collaborations, each scientist takes as a model to imitate the most successful among his collaborators rather than, for example, alternating in imitating some of them.

Regarding the neighborhood update rule, we assume first that only D-D links are broken, and second that any targeted agent always accepts a new partner. This rule can be justified as a conservative assumption when considering minimal conditions for the emergence of cooperation. As discussed in Section 4, alternative rules allowing Cooperators to sever their links would further enhance cooperation. The second assumption can be justified invoking the absence of cost to sustain a link. Within this assumption, the new link may only increase the payoff, never decrease it, thus it will always be accepted. In the

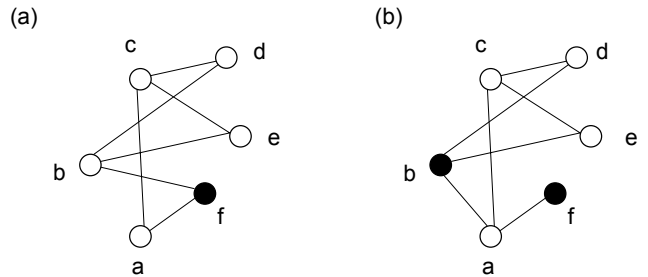


FIG. 1: (a) At time step  $t$ , the system is composed by Cooperators (white circles  $a - e$ ) and Defectors (black circle  $f$ ) whose interactions are given by the links. Given that the payoff matrix is given by  $P = S = 0$ ,  $R = 1$  and  $T = 1.3$ , the payoff for each agent is  $\{\Pi_a, \Pi_b, \Pi_c, \Pi_d, \Pi_e, \Pi_f\} = \{1, 2, 3, 2, 2, 2.6\}$ . Therefore the Cooperator  $c$  is the *leader*, because it has the maximum payoff. Agents  $a$  and  $b$  get a lower payoff than their common neighbor, the Defector  $f$ . However, agent  $a$  has another neighbor  $c$  who gets an even larger payoff than  $f$ . (b) In the next time step  $t + 1$ ,  $a$  imitates Cooperation from  $c$ , which has the largest payoff among its neighbors, and only agent  $b$  changes strategy and imitates the D-strategy from  $f$ . The figure also shows that  $b$  broke the link with agent  $f$  and starts a new interaction with the randomly selected agent  $a$ .

context of scientific collaboration, if the cost to sustain a research collaboration was zero, then scientists would accept offers from any collaborator with the expectation to get a benefit. Beyond sustaining a link, there is a cost to establish a new one. In our model, the plasticity parameter  $p$  measures how easy is both to sever a collaboration and find a new partner. While for  $p = 0$  this cost is extremely large (so large that it prevents the formation of new links), for  $p = 1$  the cost is small.

Finally, although in real social dynamics one experiences both spontaneous creation and suppression of interactions, for simplicity, our model assumes that unsuccessful relations are replaced by randomly selected new ones, leaving always the total number of links constant.

Our strategy update introduces a first classification of the agents in the network: *satisfied* and *unsatisfied* agents. An agent is satisfied if he does not imitate any other agent, so he has the largest payoff in its neighborhood; otherwise he is unsatisfied. Thus, strategy *imitation* and *social plasticity* (adaptation) is restricted only to unsatisfied agents.

The probability  $p$  measures the social plasticity of the agents, controlling the rate at which the network structure evolves, as compared to the time scale of evolution of the strategies. For values of  $p \ll 1$  strategies change much faster than network evolution (situation close to the frozen network of  $p = 0$ ), while for  $p = 1$  strategies and network evolve at the same rate (fluid social network).

### C. Stationary states

The proposed model naturally leads to a time evolution of the local connectivity of the network, featuring agents with heterogeneous neighborhoods. Starting from initial condition of random partnership with links randomly placed among initially equivalent agents, the feedback between choice of strategies and choice of partners results in a dynamical evolution from which a well-defined social structure is expected to emerge in the form of *stationary states*.

In our context a stationary state is reached when both the strategy of each agent and its neighbors remain fixed in time. Given the discrete nature of the dynamics and the finite number of possible states, these states may be reached in a finite number of time steps. The simplest stationary state consists of all agents being Cooperators. Note however, that a state composed exclusively by Defectors is a stationary state only when the network is fixed ( $p = 0$ ); for  $p > 0$ , the agents strategies remain fixed, while the network is continuously evolving. According to the dynamical rules of the model, each agent being a Defector is not a stationary configuration, even in the case of a social network where each agent has the same number of neighbors (remember than in the case of several neighbors sharing the largest payoff in the neighborhood one of them is picked at random). In an all-D network only interaction links change, but not strategies. For the sake of clarity we will refer to this state as the all-D network state, even if the interactions are not stationary. Our system has a multiplicity of different stationary states and it is expected that the system reaches one of these states. This assumption is confirmed by numerical simulations. The specific stationary state reached by the system depends on the stochastic process that shapes the network evolution. A proper characterization of the general properties of these stationary states relies then on statistical tools.

Two requirements are needed for a stationary state to exist. The first is that there are no links between two Defectors, so that the neighborhood update rule ( $p > 0$ ) does not produce any change. The second condition relates the respective payoffs in each neighborhood of any Cooperator  $i$  interacting with a D-agent, say  $d$ . Their respective payoffs must satisfy

$$\Pi_j > \Pi_d > \Pi_i, \quad (5)$$

where agent  $j$  is the Cooperator imitated by  $i$ . Thus, a stable situation occurs when the payoff of Defectors accommodate 'in-between' the payoff of two Cooperators, and they only *exploit* Cooperators. Whenever the payoff of  $d$  becomes larger than the best neighbor of  $i$ , the configuration is no longer stationary, because the Cooperator  $i$  will switch to imitate the strategy of agent  $d$ . Thus, in the stationary state, Defectors do not interact with other D-agents, and no Cooperator imitates their strategy. An interesting consequence of this result is that the agent

with the largest payoff is a Cooperator, if the system settles to a stationary state. However, this does not prevent that in a transient state a Defector can hold the largest payoff.

This simple analysis highlights the role that the 'imitation of the best neighbor', coupled to the social plasticity rule, has in shaping the social structure of the system. A related consequence is that the social structure developed becomes hierarchical in terms of payoff and in terms of the dynamics as we will show below. One way to visualize the hierarchal structure of the network is by constructing a sub-network referred as *imitation network*, where each *directed* link indicates who is imitating whom. As in stationary states no other agent imitates Defectors, in this representation, D-agents are isolated.

Figure 2 shows part of the imitation network in a stationary configuration. The agents in the outer layers imitate the ones in the inner layers: by our previous definition they are unsatisfied agents. In a stationary state, Cooperators form *trees* of agents hierarchically ordered according to their payoff. This description highlights some special agents that do not imitate any other agent: the agent in the top of the chain has the highest payoff of the tree it belongs and he is the only agent that is satisfied in the tree. They are easily recognized at the top of the trees. All other agents in the tree are unsatisfied, but they imitate the same strategy they were playing in the previous time step. An implication of this representation is that the size of the *group* formed by following directed links to the leader, gives an indication of the influence of the leader with respect to the rest of the system. In a stationary state, a leader has no links to D agents. Among the leaders there is an absolute leader defined as the one with the largest payoff in the system. As a consequence it is also the agent with the largest number of links in the imitation network. All the links of the absolute leader are imitation links shown in the imitation network; any other leader in the system has a smaller number of cooperating partners. The whole system is dominated by the few leaders that control a large fraction of the population. The structure of the social network is then expected to be very sensitive to perturbations acting on those leaders.

When the system is close to a stationary state (i.e., if there are a few Cooperators not satisfying Eq. (5), as it happens when, for example, a Cooperator imitates a Defector), then the imitation network is useful to understand how this 'perturbation' will propagate along the rest of the social structure. Note that at each time step, the D strategy replicates on all those agents connected to the agent where the perturbation started, causing an 'avalanche' of replication events. This chain of events may end in a all-D network or due to the existence of Cooperator leaders, cooperation may recover, as we will show below.

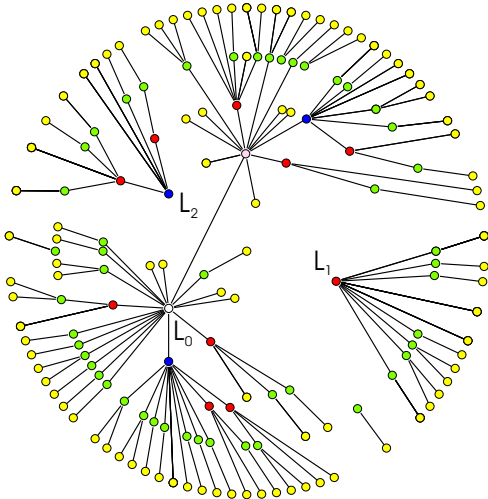


FIG. 2: Imitation network obtained from a numerical simulation with  $T = 1.75$ ,  $R = 1$ ,  $P = S = 0$  and  $p = 0.10$ . The simulation starts from a random network with  $N = 10000$  agents and a number  $K = 8$  of average links per node of the network. Agents are organized in imitation layers coded with the same grey level. Starting from the outer layer of a tree, new nodes are introduced for each new link in the path to reach the leader at the top of the tree. Each agent imitates the agent of the inner layer to whom it is linked in the tree. The network contains several trees. Three leaders ( $L_0$ ,  $L_1$  and  $L_2$ ) are identified.  $L_0$  is the absolute leader with the largest payoff in the system. Its tree contains approximately half the number of agents of the system. All nodes are Cooperators, because Defectors are isolated in the imitation network. Most of the network's nodes, which belong to the lowest layer of the hierarchical structure, are not shown for clarity.

#### D. Social differentiation

Our previous description of the resulting stationary states, offers also a description of how our social model with network adaptation naturally gives rise to a process of social differentiation, with the spontaneous emergence of different social roles. Even if all the agents are driven by the same dynamical rules and they are initially statistically equivalent, their role in the network diversifies. Our previous analysis of the imitation network allows us to identify three types of agents in a stationary state:

- **Leaders:** satisfied Cooperators that have the maximum payoff in their corresponding neighborhood. The absolute leader is the agent with the largest payoff in the whole network, and its corresponding group of influenced agents is in general the largest. Typically the other leaders have also a large number of links and a payoff above the average. Admittedly, the sociological concept of leader has several different characteristics, not all of them included in our definition. Still we use this term to empha-

size that those are agents strongly influencing other agents to adopt their strategy. In this context leaders do not have the will of seeking their leadership nor do they try to preserve it.

- **Conformists:** unsatisfied Cooperators, i.e. they do not have the maximum payoff in their neighborhood, but they imitate another agent playing their same strategy. They constitute the large majority of the nodes of the imitation network except for the leaders.
- **Exploiters:** Defectors who take advantage of other's actions. Defectors have a larger payoff than their (C) neighbors, showing they succeed in exploitation, and are satisfied. In the imitation network they do not have any directed link, because in the stationary state no other agent imitates their strategy.

### III. SIMULATING SOCIAL DYNAMICS

To facilitate comparison with previous results of the PD game in fixed networks (e.g., (Nowak and May, 1992)), we have performed numerical simulations of the above model using as main parameters the incentive to defect  $b \equiv T$  in the range  $1 < b < 2$  and the social plasticity  $p \in [0, 1]$ . The rest of the PD payoff matrix parameters have been kept fixed,  $R = 1$ ,  $P = 0$ , and  $S = 0$  as in Nowak and May (1992). It has been previously found (Lindgren and Nordahl, 1994) that turning  $P = S = 0$  does not change the main results, although the single PD game is not in the strict Prisoner's Dilemma conditions. We have checked that changing the parameter  $P$  in the region  $(0, 0.1)$  does not affect significantly our results.

All simulations were performed with  $N = 10000$  agents. The strategy initial condition was always set to 60% of randomly distributed Cooperators. The network initial condition was set by distributing  $KN/2$  undirected links between random pairs of nodes. The number  $K$  corresponds to the average connectivity per agent, and we considered  $K = 8$ . The initial degree (number of links of each agent) distribution constructed in this manner is a Poisson distribution with parameter  $K$ . Simulations with other values parameters have been performed and no qualitative differences were found. Likewise we have checked that using an asynchronous updating in our simulations does not change any meaningful qualitative result. For the above parameters, a stationary state is reached in a time scale that depends very much on  $p$ , requiring about a 1000 time steps for  $p = 0.01$  and about 10 time steps for  $p = 1$ . For smaller  $N$  the typical time to reach a stationary state decreases, while for larger  $K$  it takes longer time. A proper characterization of the general properties of these stationary states relies then on averaging over many different numerical experiments. Our results are given as averages over 100 different experiments.



Simulations in our adaptive dynamic network game show how Cooperation is in general enhanced and no threshold to an all-D network is observed. In fact, our results indicate that for a given set of parameters, there is a *coexistence* of a multitude of cooperative stationary states and the all-D network state. This means that for a fraction of the initial conditions, the stochastic dynamics of the network may lead either to the all-D network, or to a stationary state. Once in the all-D network, the system gets trapped and no recovery is possible (remember that in an all-D state the interaction links change, but not strategies). The probability of reaching the trapping all D-network, decreases with increasing the number of agents (it also decreases with decreasing the incentive to defect  $b$  and increasing the initial distribution of Cooperators), and thus the trap appears to be a finite size effect. For instance, for an incentive to defect  $b = 1.6$  and plasticity  $p = 0.01$ , only 6% of the realizations get trapped for a large population of  $N = 10000$ , while in contrast for  $N = 1000$ , 80% get trapped. Here we concentrate on stationary states where Cooperators coexist with Defectors.

### A. Fraction of Cooperators

Figure 3 shows a first global characterization of the stationary states, for different values of the parameter  $p$  and the incentive to defect  $b$ . The fraction of Cooperators,  $f_C$ , is measured for a fixed network ( $p = 0$ ) and is shown to decrease with  $b$ , approaching 0 at a threshold value of  $b \simeq 1.75$ . Thus context preservation (Cohen et al., 2001) without social plasticity provides partial cooperation. In clear contrast, for positive  $p$ , social plasticity facilitates the establishment of a highly cooperative state with a fraction of Cooperators essentially independent of the incentive to defect  $b$ . Even for small plasticity  $p$ , the fraction of Cooperators is above 90% in the range of parameter  $b$  considered. A similar result is obtained for other values of the average connectivity  $K > 2$ . In this sense, context preservation is not a necessary condition to build up cooperation, but rather the social structure is a consequence of an adaptive dynamics in which cooperation is greatly enhanced.

It is worth mentioning that the result of enhanced cooperation works against the direct effects of the neighborhood adaptation rule, introduced to facilitate the survival of Defectors. Defectors are allowed to find new Cooperators to exploit, exiting from unsatisfactory interactions with other Defectors. However, the final outcome is that the number of Defectors decreases. It seems that successful Defectors become isolated because they are role models: their victims “run away”. The new Defectors – those that imitated a successful Defectors – establish links with some Cooperators who have a high concentration of some other Cooperators in their neighborhoods. Thus the formerly exploited Cooperators, now Defectors, turn again into Cooperators. What it is needed to get cooper-

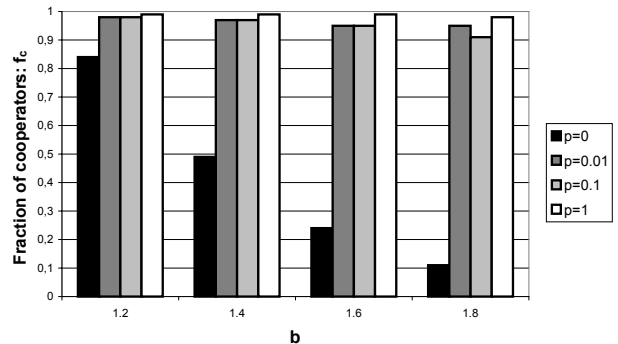


FIG. 3: Average fraction of cooperative agents,  $f_C$ , for different values of the plasticity  $p$  and incentive to defect,  $b$ . For a fixed network ( $p = 0$ ),  $f_C$  decreases with  $b$ . However in the presence of social plasticity ( $p \neq 0$ )  $f_C$  is kept above 90%.

ation is Cooperators with enough Cooperator partners to have a payoff higher than any Defector. This hypothesis is analyzed in the following sections.

### B. Distribution of Cooperators' and Defectors' payoffs

The total average payoff is larger in the dynamic network ( $p \neq 0$ ) than for a frozen network ( $p = 0$ ). However, we have measured systematic differences when considering the average payoffs  $\Pi_D$  and  $\Pi_C$ , for each of the respective sub-populations D and C. While the number of Cooperators is larger, their average payoff is smaller than the one of the Defectors, reversing the situation of what happens in a frozen network (see Fig. 4). The payoff distribution in a stationary state for each subpopulation is shown in the inset of Fig. 4 for a particular value of  $b$ . This graph shows that the most probable payoff is larger for Defectors, explaining also the larger average payoff of the Defectors. However, close inspection reveals that the distribution for Cooperators has always a larger tail, indicating that there is a number of Cooperators with a payoff larger than any other Defector in the network. Most of these agents belong to the class of the leaders, and are necessary for a cooperative stationary state to exist.

Thus the main characteristics of our adaptive social game, is that an altruistic social network, i.e., a network composed by a large fraction of Cooperators, may develop with few Defectors but which have a larger average payoff. The neighborhoods of the agents adapt to conform the requirements of the stationary states (D-agents only interact with Cooperators), and the Cooperator with a largest payoff in each branch of the imitation network corresponds to a ‘leader’, with a payoff much larger than the average, from which all other agents imitate its strategy.

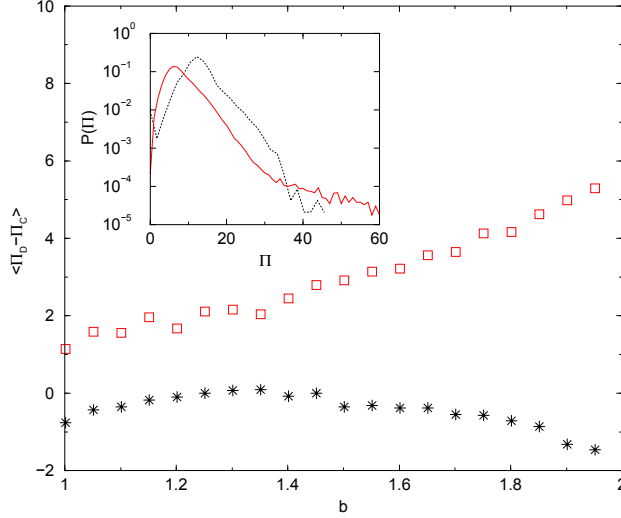


FIG. 4: Difference between the average payoff of Defectors,  $\langle \Pi_D \rangle$ , and Cooperators,  $\langle \Pi_C \rangle$ , as a function of  $b$  for  $p = 0$  (stars) and  $p = 0.10$  (squares). The effect of the social plasticity is that the Defectors get a larger payoff on average:  $\langle \Pi_D - \Pi_C \rangle$  is positive for  $p \neq 0$ . Inset: probability distribution (normalized to each sub-population) of individual payoff, for Cooperators (solid line) and Defectors (dotted line). The most probable payoff, which corresponds to the maximum of the distribution, is larger for Defectors than for Cooperators. However, there are a few Cooperators getting the largest payoff in the population as it is seen in the tails of the distribution for large values of  $\Pi$ . Parameter values  $b = 1.75$ ,  $p = 0.1$ .

### C. Transient dynamics, leaders and social crisis

The evolution towards a stationary state is typically not a smooth monotonous build-up of a globally cooperating state. Starting from a random initial condition, the transient dynamics is characterized by small fluctuations and some distinctive large oscillations in the fraction of Cooperators as a function of time, that can be understood in terms of *social crisis*.

An example of the frustrated attempts to built cooperation is given by the evolution of the fraction of Cooperators shown in Fig. 5. Starting from a non-stationary state of low cooperation in the initial fixed random network for  $t < 200$ , network dynamics then leads to a social network with a high degree of cooperation after several large oscillations in the time interval  $220 < t < 300$ . The large oscillations in  $f_C$  are frustrated attempts to build cooperation. This indicates that the defecting behavior is so rewarding, that the cooperation has to find a specific network configuration in order to be robust against eventual changes of strategy. In such configuration the most connected agent (largest number of links) in the imitation network is also the one with largest payoff. In the frustrated attempts to reach a global stationary cooperative state, the fraction of Cooperators becomes large. A few time steps before  $f_C$  reaches each maxima, a Co-

operator receives a new link from a Defector with larger payoff, and switches to D-strategy, starting a social crisis. Given that at those time steps there is a large proportion of Cooperators in the network, a fast avalanche of imitation of D-strategy starts. This imitation will certainly affect the neighbors of a Cooperator leader, thus decreasing the payoff of most leaders. Precisely when an avalanche starts, there is a Defector with a payoff *larger* than the *absolute Cooperative leader*. The social crisis propagates through most of the system, producing a change in the connectivity of the social network (Zimmermann et al (2001)). The initial distribution of the number of links for the C and D populations are Poisson distributions around the average connectivity, typical of random networks. At the time of a local maximum of  $f_C$  the two distributions have exponential tails for large values of the number of links. Then very rapidly after a local maximum of  $f_C$ , the network switches to the almost defective solution with a large number of D-D links and Poissonian degree distributions. However the existence of a small number of Cooperators with a large payoff permits, thanks to the plasticity of the network, the gradual build-up of cooperation by creating C-C links. This requires two steps. First D-C links are created (initiated by the Defectors) and then the Defectors imitate the more successful Cooperator in a later stage. Finally in the stationary state, the distribution of the number of links for the D population becomes very narrow, while the distribution for the C population displays a tail approaching an exponential decay. The stationary network configuration is thus dominated by a few Cooperators – the leaders – with a large number of links (the tails of the distribution of the number of links for Cooperators). These highly connected agents dominate the collective behavior of the network.

This characteristic dynamics reveals the *functional property* of C-leaders; on one hand, due to the imitation process they sustain cooperation, while on the other they enhance cooperation whenever the C-leader has the largest payoff of the whole network. In fact there is a sort of competition between the Cooperative leader and the Defector with largest payoff. We remark that the latter process is due to the social plasticity: whenever a Defector selects a leader for partnership, there is a large probability that it has a larger payoff and the Defector will imitate cooperation, *enhancing* the number of Cooperators.

Figure 5 also illustrates the sensitivity of the stationary network structure to exogenous perturbations acting on the leaders, which reflects their key role in sustaining cooperation. At time  $t = 500$ , the system already reached a stationary state. However, at this time step, we have forced a strategy switch of the cooperative absolute leader, leading once again to a social crisis similar than the previously described in the endogenous dynamics. In summary, the system self-organizes itself in one of several possible cooperative states where avalanches and social crisis are likely to occur following spontaneous

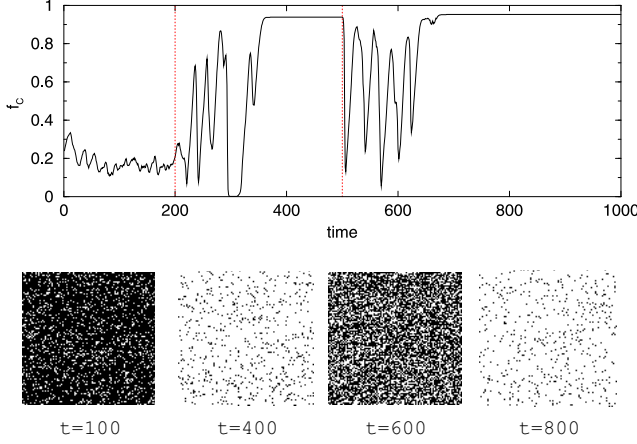


FIG. 5: Time series for the fraction of Cooperators  $f_C$  for  $b = 1.7$ . The evolution is in a fixed random network ( $p = 0$ ) up to time  $t = 200$  when network dynamics is switched-on, so that  $p = 1$  for  $t > 200$ . At time  $t = 500$  the network leader is forced to change action from C to D, to show how cooperation may recover after a social crisis. After an oscillatory transient, a new stationary state is reached. Below, snapshots of the instantaneous configurations of actions are shown at the indicated times. A white (black) point is a C (D) agent in a node of the network (links are not shown). The spatial location of the agents does not have any meaning due to the building of a network of interactions. It is only used for displaying purposes.

focused local perturbations.

#### D. Structure of the social network

A first characterization of the topology of the stationary network reached by the dynamics is given by the degree distribution, i.e., the probability distribution of the number  $K$  of links of a node. This distribution has a long tail that distinguishes it from the Poisson distribution of the random network. This is reflected in the values of the normalized standard deviation  $\sigma_n = \sigma/K$  shown in Fig. 6 for different values of  $p$  and  $b$ . We recall that for a Poisson distribution  $\sigma_n = K^{-1/2}$ , while for an exponential distribution  $\sigma_n = 1$ . We find that for small values of  $b$  the distribution departs significantly from the Poisson distribution only for large values of the plasticity parameter  $p$ , while for increasing  $b$  the tail of the distribution expands and approaches an exponential form. In other words, the hierarchical structure of the network accentuates as  $b$  increases, with fewer leaders that have a larger payoff.

The distribution of payoffs (inset of Fig. 4) is similar to the degree distribution, because the payoff of the Cooperators is given by the number of links with other Cooperators, and there is a very small number of Defectors in the stationary state. Therefore, the variations

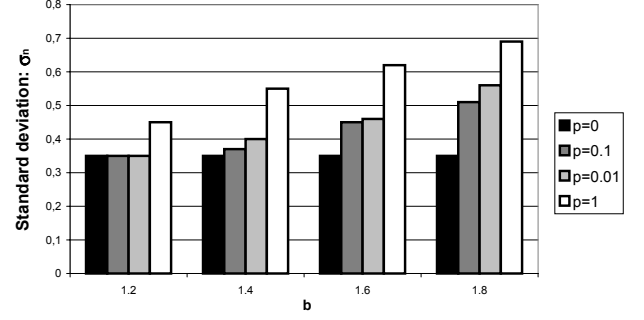


FIG. 6: Normalized standard deviation  $\sigma_n$  of the degree distribution of the social networks obtained for different values of the plasticity parameter  $p$  and the incentive to defect  $b$ . For a Poisson distribution and  $K = 8$ ,  $\sigma_n = K^{-0.5} = 0.35$ , while for an exponential distribution  $\sigma_n = 1$ . For  $p = 0$ ,  $\sigma_n$  takes the value of a Poisson distribution. As  $p$  and  $b$  are increased the interaction network departs from a Poisson distribution and approaches a distribution with an exponential tail.

of  $\sigma_n$  also provide a measure of social inequality in the network. In fact  $\sigma_n$  is closely related to the Gini coefficient (Kakwani, 1980) used in the characterization of economic inequalities in a social group. We have measured the Gini coefficient of the resulting network configuration in a stationary state, and found that it can be twice as large compared to a random (Poissonian distribution) network. Social plasticity generates a flux of payoff towards richer individuals.

Finally, we address the question of whether the social structure generated in our dynamical model has the characteristics of Small World network. Two requirements have to be fulfilled: the clustering (or cliquishness) has to be much larger than in a random network, and the average path length between two nodes should be similar to the one of a corresponding random network. The clustering coefficient,  $c$ , measures the fraction of neighbors of a node that are connected among them, averaged over all the nodes in the network. Most real complex networks show a clustering larger than in random networks given by  $c_{rand} = K/N$  (Amaral et al., 2000). In our original formulation, numerical simulations show (Fig. 7) that for increasing  $b$  the clustering coefficient increases very mildly with respect to the clustering of a fixed random network (i.e.  $c/c_{rand}$  may change up to 1.06). We have tested a slight enhancement of the network adaptation, which easily accounts for high clustering. Very often, new acquaintances are made based on the relationships of current neighbors. To implement this idea, the social neighborhood adaptation step of our original formulation was augmented: if a neighbor is replaced by a new one, with probability  $q$  a *local* selection of a new partner is done among the neighbors of the neighbors, while with probability  $1 - q$  the previous random selection is performed. The limiting case  $q = 0$ , corresponds to our original formulation, while  $q = 1$  corresponds to the case that all

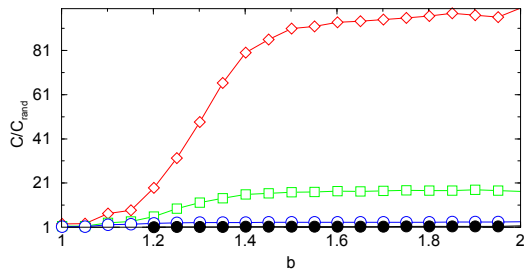


FIG. 7: Normalized clustering coefficient,  $c/c_{rand}$ , with  $c_{rand} = K/N$ . Filled circles correspond to  $p = 1$  and  $q = 0$ . Other curves are for  $q = 0.01$  and  $p = 0.01$  (diamonds),  $p = 0.1$  (squares), and  $p = 1$  (empty circles). Social plasticity ( $p \neq 0$ ) and a local selection of new partners ( $q \neq 0$ ) is needed in order to reproduce the clustering observed in social networks.

the new partners are chosen from the neighbor's neighbors. It is natural that this new mechanism will increase the clustering. Numerical simulations show that while most of our results previously discussed are qualitatively independent of the value of  $q$ , with a very small value of  $q$  the clustering coefficient reaches a very large value. For instance, 1% of local partner selection is enough to increase  $c$  a hundred times, being the clustering largest for a slow evolution of the network ( $p \ll 1$ ). For the second requirement for small world topology, we find that the average path length remains in all cases very close to the one of a random network. All together our results indicate that allowing for local partner selection, the social network generated in our adaptive dynamics has the structure of a small world network.

#### IV. DISCUSSION

We have presented a minimal model which incorporates simultaneous and coupled evolution (*co-evolution*) of the strategies of the agents and of the social network, providing a first step in the investigation of the processes of social differentiation in a globally cooperating social group. Different agents end up playing different social roles. These roles are acquired through social interaction and they are not externally imposed or determined by genetic mechanisms. Rather, the roles *emerge* from the self-organizing dynamics of the complex system. It is here important to note that the initial differences in number of links among the agents do not determine the final role of each agent, since each agent changes temporarily its role in the stochastic dynamics until a final stationary state is reached. Moreover, we have checked that taking as initial condition a random network in which each agent has the same number of links, the system dynamics leads to the same emergent role differentiation.

Our results indicate that cooperation is stabilized by a

hierarchical self-organized structure, so that the formation of cohesive clusters of Cooperators with exclusion of free riders is not needed for persistent global cooperation. Allowing for some local partner selection, this self-organization leads from a random network to a final social network with the topology of a Small World network, giving an example of how the small world connectivity, in which clustering is larger than in a random network, can be dynamically achieved.

Our study reveals that if Defectors have the ability to choose partners, breaking interactions with other Defectors, the number of Cooperators increases, but the average payoff of Cooperators is less than that of greedy agents. The dynamics naturally generates *leaders* - individuals getting a large payoff who are imitated by a considerable fraction of the population-, *conformists* -unsatisfied cooperative agents that keep cooperating-, and *exploiters* -Defectors with a payoff larger than the average one obtained by Cooperators. The most prominent role is the one of leader, a Cooperator which not only sustains cooperation, but, it also drives the whole system towards more cooperation. Defectors are found to remain in a stable situation whenever they *exploit* Cooperators. The formation of a social hierarchy in the population is the source of possible unstable behavior. It promotes the occurrence of social crisis that can affect a large fraction of the population. These crisis take the form of global cascades that might be easily triggered by the spontaneous change of the action of a highly connected agent. This result identifies the importance of highly-connected agents that, as illustrated by the imitation network, play a leadership role in the collective dynamics of the system. Such sensitivity of the network stability to local special perturbations provides an interesting feature of globalization: the group of agents organizes itself in a state where an exogenous or stochastic perturbation may produce drastic changes, at distance, in a finite time.

Even though our model shows many interesting aspects, the strategy and neighborhood update rules represent an extreme and conservative choice of rules. Thus, it would be important to investigate how the modification of the model rules affect the results reported here. Some of the points that we consider that merit further research in connection with our *strategy update* and *network update* rules are indicated in the remainder.

Starting with our strategy update rule, we note that an important assumption made in our model is that the satisfaction of an agent is determined by comparison of absolute payoffs. Due to the evolution of the social interaction this implies that two neighbors might have different payoff just because they have a different number of neighbors. On the one hand this incorporates the idea of the importance of being highly connected. On the other hand, the possibility of using a comparison based on the relative payoff per neighbor would imply that each agent knows how many neighbors has each of its neighbors. Additionally, our strategy update rule is an extreme “copy best” rule, while alternatives of probabilistic imitation of

better role models (Schlag and Pollock, 1999), or probabilistic selection of neighbors strategy (Nowak and May, 1993) could also be considered.

Turning now to the network update rule, we note that in our model only links between two Defectors can be possibly broken. In a stationary state most Cooperators are unsatisfied because they imitate an agent with a larger payoff. In our model this unsatisfactory situation does not induce an action to improve their payoff beyond the imitation of a neighbor in the social network. One way to test the implications of this hypothesis would be modifying the network update rule, letting the C-D link also to be broken, for example letting Cooperators have some probability to change a D-neighbor by a random agent. It is clear that in this setting cooperation would be further enhanced, and it could also limit the occurrence of large social crisis. Thus, our rule for neighborhood update assures that particularly the highly successful Defectors won't be abandoned by the Cooperators whose exploitation has made them so successful. Hence, we make Defection an attractive role model because exploited agents can not simply leave their exploiter, e.g., due to costs of relationship change, or some (semi-)rational calculation of the expected benefit of creating new ties. A related point is that one could question why not all D-D links have the possibility of being broken and not only those that involve an unsatisfied agent. We have in fact considered that only agents having at least one neighbor with higher payoff, are "forced" to do something to improve their payoff. Thus, we assume some kind of "aversion to change": do something only when you are not satisfied, otherwise do nothing. This rule can be justified invoking some cost implicit in the change of a social relation, and it allows, in principle, to keep the interaction between two Defectors as long as they are satisfied.

There is another point which deserves further investigation and which relates both to the strategy and network update rules: Our choice of strategy update is based on the aggregate payoff, while our justification of the network adaptation rule is based on comparison of individual actions. This a priori involves two different mechanisms and it is clear that our choice is one among sev-

eral possible. We have already mentioned a strategy update (Nowak and May, 1993) based on probabilistic selection among the neighbors strategy, weighted by the aggregate payoff. Cohen et al. (2001) compares different such mechanisms on fixed networks. On the other hand, fewer results are known on network update mechanisms. In the economics literature a common choice is that a link among two economic agents is accepted if *both* agents improve their payoff (Bala and Goyal, 2000; Jackson and Watts, 2002). It would be interesting to test new strategy and network adaptations involving a more sophisticated evaluation of the strategy of the opponent and its aggregate payoff. One possible direction is to contemplate a more complex strategy space, involving strategies dependent on the past encounters. Work in this direction with fixed social network was initiated by Lindgren and Nordahl (1994). Another interesting direction is to consider non-equivalent agents, so that they may differ in their attractiveness as exchange partners. Flache (2001) has incorporated this situation in a model that combines the agents' decisions about cooperation with the decisions about selection of new partners. They also obtain a process of social differentiation sustaining cooperation. It is unclear how much of the process of social differentiation originates in the unequally attractive agents or on the simpler mechanisms contained in our model.

Beyond our update rules, the addition of random perturbations in strategy and network is also a very relevant feature to be explored in the future. These perturbations may originate from errors in imitation or payoff determination, for example. Here we have just shown that strategy perturbations may cause large social crisis, specially if a well connected agent makes an error.

We finally remark that many contexts in human societies follow the dynamic scheme considered here – new collaborations are frequently formed, while other long lasting partnerships die out. From scientific collaboration to sports teams –political parties not to be forgotten–, our study offers an example of simple mechanism by which leadership, and also other social roles, might appear and consolidate.

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